

Year 11 Mathematics Specialist Units 1,2
Test 1 2021

Section 1 Calculator Free
Counting Techniques, Vectors

STUDENT'S NAME _____

DATE: Thursday 4 March

TIME: 20 minutes

MARKS: 23

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (6 marks)

(a) Determine an expression for the number of arrangements of all letters for each of the following. Do not simplify.

(i) Chemistry $9!$ [1]

(ii) Repetitions $\frac{11!}{2! \cdot 2! \cdot 2!}$ [2]

(b) Solve $\binom{n}{5} = \binom{n}{7}$ [3]

$$\frac{n!}{5!(n-5)!} = \frac{n!}{7!(n-7)!}$$

$$\frac{7!}{5!} = \frac{(n-5)(n-6)(n-7)!}{(n-7)!}$$

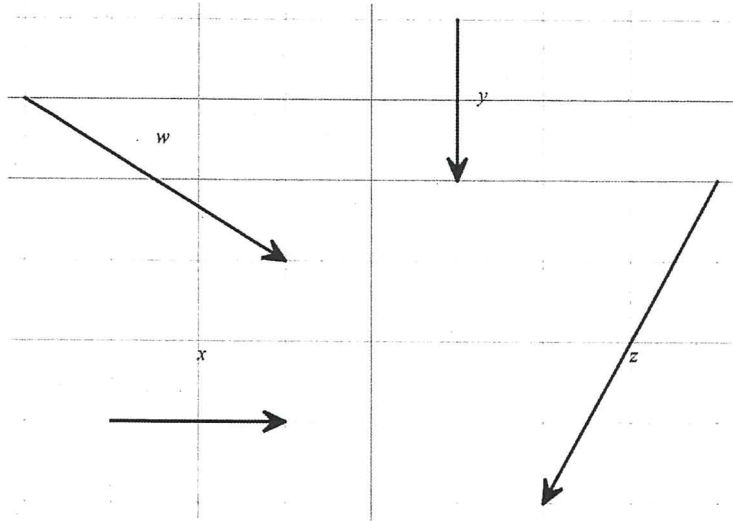
$$42 = n^2 - 11n + 30$$

$$0 = n^2 - 11n - 12$$

$$0 = (n-12)(n+1)$$

$$n = 12, \quad \cancel{-1}$$

2. (7 marks)



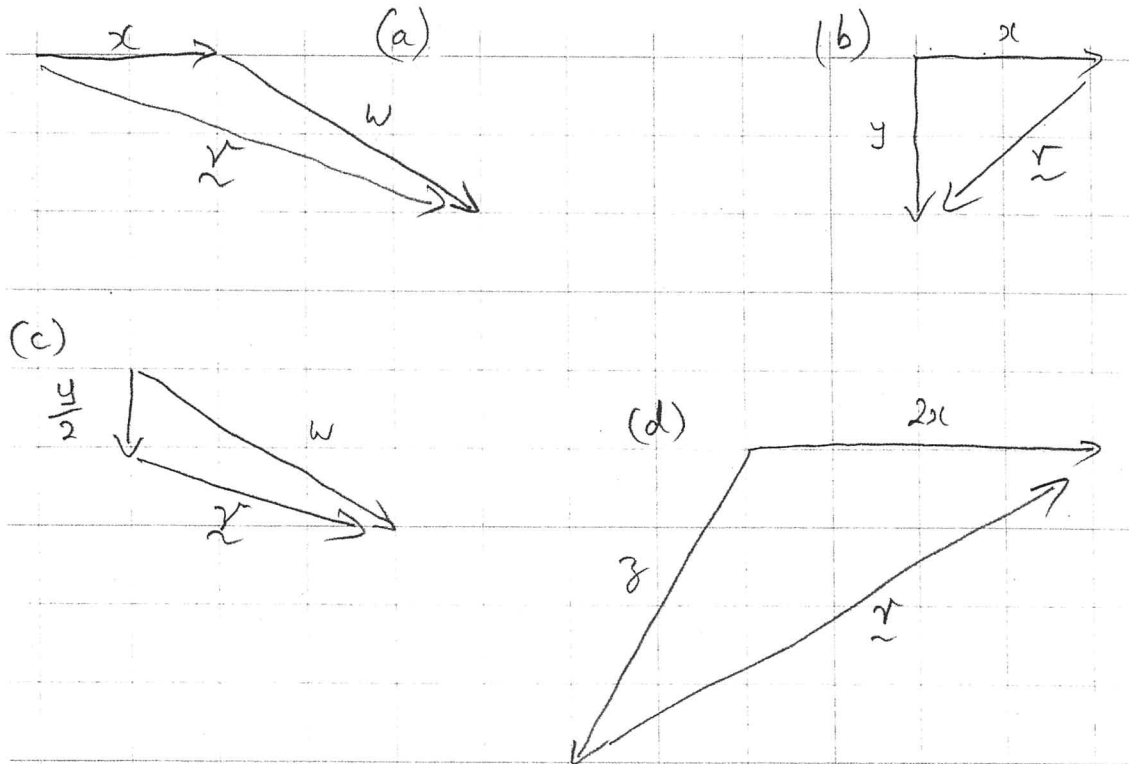
On the grid below, draw the resultant of each of the following. Label each diagram.

(a) $x + w$ [1]

(b) $y - x$ [2]

(c) $w - \frac{y}{2}$ [2]

(d) $-z + 2x$ [2]



3. (10 marks)

A combined sporting club committee is formed with the number on the committee from each sport proportional to each sports' number of players. There are 8 from football, 7 from hockey and 6 from soccer. A special sub-committee of 7 is to be formed from this committee. With answers being left in combination or factorial notation, determine how many ways can this special sub-committee be formed if;

(a) there are no restrictions on selection [2]

$$\binom{21}{7}$$

(b) there is to be 3 from football, 3 from hockey and 1 from soccer [2]

$$\binom{8}{3} \times \binom{7}{3} \times \binom{6}{1}$$

(c) due to a disagreement, Myles and Alan will not serve together on the sub-committee [3]

$$\binom{2}{0} \binom{19}{7} + \binom{2}{1} \binom{19}{6}$$

(d) there must be at least 2 members from each sport [3]

$$\binom{8}{3} \times \binom{7}{2} \times \binom{6}{2} + \binom{8}{2} \times \binom{7}{3} \times \binom{6}{2} + \binom{8}{2} \times \binom{7}{2} \times \binom{6}{3}$$

Year 11 Mathematics Specialist Units 1,2
Test 1 2021

Section 2 Calculator Assumed
Counting Techniques, Vectors

STUDENT'S NAME _____

DATE: Thursday 4 March

TIME: 30 minutes

MARKS: 31

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

4. (4 marks)

Given \tilde{a} and \tilde{b} are non-parallel vectors and that λ and μ are scalar quantities, determine the value of λ and μ when;

$$2\lambda\tilde{a} - 11\tilde{a} + 5\mu\tilde{b} = \mu\tilde{a} - 3\lambda\tilde{b} - 3\tilde{b}$$

$$2\lambda a - 11a - \mu a = -3\lambda b - 5\mu b - 3b$$
$$a(2\lambda - \mu - 11) = b(-3\lambda - 5\mu - 3)$$

$$2\lambda - \mu - 11 = 0$$
$$-3\lambda - 5\mu - 3 = 0$$

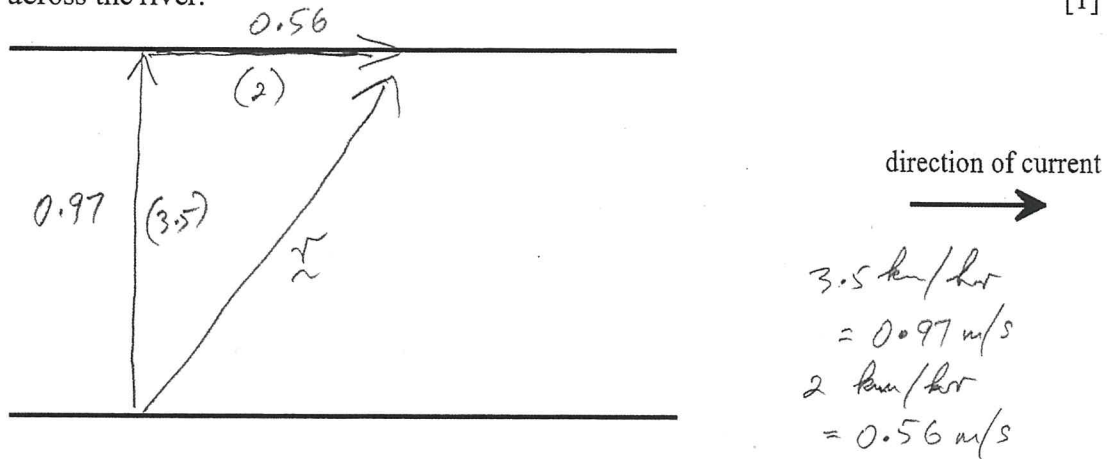
$$\lambda = 4$$

$$\mu = -3$$

5. (8 marks)

A section of a river has parallel banks and is 270 m wide. Aaron sets out to swim directly across the river. He can swim at 3.5 km/hr and the current is flowing at 2 km/hr.

- (a) On the diagram below show what happens to Aaron when he attempts to swim directly across the river. [1]



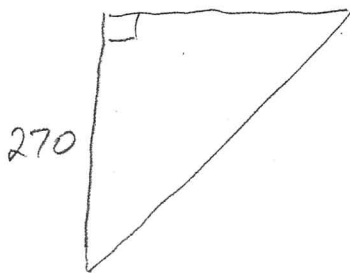
- (b) Calculate Aaron's actual velocity. [2]

$$\sqrt{2^2 + 3.5^2} = 4.03 \text{ km/hr}$$

$$\sqrt{0.97^2 + 0.56^2} = 1.12 \text{ m/s}$$

(1.12 m/s)

- (b) Determine how long it will take Aaron to reach the other side of the river and how far he will end up from the point directly across the river when he reaches the river bank on the other side. [4]



$$T = \frac{D}{S}$$

$$= \frac{270}{0.97}$$

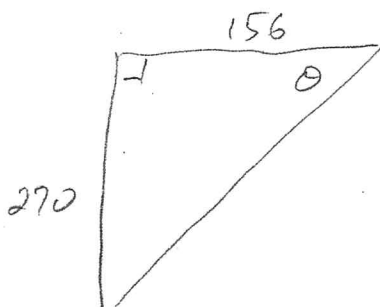
$$= 278 \text{ sec}$$

$$D = S \times T$$

$$= 0.56 \times 278$$

$$= 156 \text{ m}$$

- (c) At what angle to the river bank does Aaron travel? [1]



$$\tan \theta = \frac{270}{156}$$

$$\theta = 60^\circ$$

6. (10 marks)

Using only the digits 0, 1, 2, 3, 4, 5, and without repetition, determine how many different numbers can be formed if:

(a) the numbers are all 3 digit numbers greater than 450. [2]

$$\begin{array}{c}
 \begin{array}{|c|c|c|} \hline 4 & 5 & \\ \hline 1 & 1 & 3 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline 5 & & \\ \hline 1 & 5 & 4 \\ \hline \end{array} \\
 3 + 20 \\
 = 24
 \end{array}$$

(b) the numbers must be divisible by 5 [4]

$$\begin{array}{c}
 \begin{array}{|c|} \hline 0,5 \\ \hline 2 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 5 & \\ \hline 4 & 1 \\ \hline \end{array} + \begin{array}{|c|c|} \hline 0 & \\ \hline 5 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline 5 & & \\ \hline 4 & 4 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline 0 & & \\ \hline 5 & 4 & 1 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|} \hline 0 & & & \\ \hline 5 & 4 & 3 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline 5 & & & \\ \hline 4 & 4 & 3 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|} \hline 0 & & & & \\ \hline 5 & 4 & 3 & 2 & 1 \\ \hline \end{array} \\
 + \begin{array}{|c|c|c|c|c|} \hline 5 & & & & \\ \hline 4 & 4 & 3 & 2 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|} \hline 0 & & & & & \\ \hline 5 & 4 & 3 & 2 & 1 & 1 \\ \hline \end{array} + \begin{array}{|c|c|c|c|c|c|} \hline 5 & & & & & \\ \hline 4 & 4 & 3 & 2 & 1 & 1 \\ \hline \end{array}
 \end{array}$$

$$2 + 4 + 5 + 16 + 20 + 60 + 48 + 120 + 96 + 120 + 96$$

(c) the numbers must be less than 2000 [4]

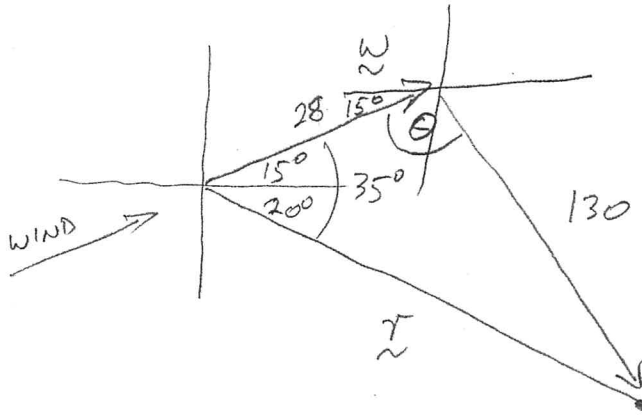
$$\begin{array}{c}
 \begin{array}{|c|} \hline 1 \\ \hline 6 \\ \hline \end{array} + \begin{array}{|c|c|} \hline & \\ \hline 5 & 5 \\ \hline \end{array} + \begin{array}{|c|c|c|} \hline & & \\ \hline 5 & 5 & 4 \\ \hline \end{array} + \begin{array}{|c|c|c|c|} \hline 1 & & & \\ \hline 1 & 5 & 4 & 3 \\ \hline \end{array} \\
 6 + 25 + 100 + 60
 \end{array}$$

$$6 + 5 + 30 + 20 + 120 + 12$$

7. (9 marks)

A small plane can fly at 130 km/hr in still air. The pilot intends to fly to a nearby airstrip on a farm located 96 km away on a bearing of 110° . Throughout the flight there is a wind blowing at a steady 28 km/hr from 255° .

(a) Determine the course the pilot should set in order to travel directly to the nearby airstrip. [5]



$$\theta = 137.9^\circ$$

$$\begin{aligned} \text{BEARING} &= 270 - 15 - 137.9 \\ &= 117.1^\circ \text{ T} \end{aligned}$$

(b) What is the ground speed of the plane during the flight? [2]

$$|\vec{T}| = 151.9 \text{ km/hr}$$

(c) What is the time difference caused by the wind? [2]

$$S = \frac{D}{T}$$

$$T = \frac{D}{S}$$

$$\begin{aligned} T &= \frac{96}{130} \times 60 \\ &= 44.3 \text{ MINS} \end{aligned}$$

$$\begin{aligned} T &= \frac{96}{151.9} \times 60 \\ &= 37.9 \text{ MIN} \end{aligned}$$

$$\text{DIFF} = 6.4 \text{ MIN}$$